“Splitting the pie”: Sharing fairly in the context of market exchanges between buyers and sellers

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Introduction

A resource may be shared differently, depending on whether the resource is divisible or not, and whether aggregating shares can bring significant economies of scale or not.

This paper focuses on a specific sharing approach called “splitting the pie,” which is usually applied to divisible resources if, for various reasons, the parties prefer not to aggregate their shares. This can happen if aggregation would not lead to larger returns to scale, or if the use of the resource precludes aggregation.

The most fundamental of market transactions is the transaction between buyers and sellers, which involves sharing the gains from trade by splitting it between them. This paper shows that the concepts of sharing and fairness lie at the very core of market transactions, requiring both sides of the transaction to reveal their deepest values—ideology, if you will—as they agree (or not) to split the gains of trade between them.

This paper further examines how the split proceeds under several perspectives—neoclassical price theory, game theory, various philosophical theories of justice, and the deeply-held values of most societies.

Sharing the gains from trade

When buyers and sellers face off in the market, their willingness (or reluctance) to buy or to sell depends on the offered price relative to their respective reservation prices. If the price is above the reservation price of the buyer, he opts out of the market. If the price is below the reservation price of the seller, he opts out of the market. Otherwise, the difference between their reservation prices is the potential gain from trade that can accrue to them together. The final transaction price determines how they will split between them the shared gain from trade.

The neoclassical analysis of supply and demand

Consider the simplest (i.e., linear) neoclassical representations of demand and supply: $p = R – D \bullet q$ (demand), and $p = C + S \bullet q$ (supply), where p stands for price, q for quantity, D and S for the slopes of the demand and supply functions respectively on the q-p plane (representing their reluctance to conclude the transaction), and R and C for the reservation prices of the buyer and the seller respectively. Neoclassical price theory treats the two as a system of simultaneous equations. Solving for q gives:

$$q=\frac{R-C}{S+D}$$

The difference R – C is the *gain from trade*, the amount that the two parties stand to jointly gain, if they conclude the trade. The greater this difference, the greater the incentive for the two sides to trade. The shared gain from trade is the "pie" that they will split at the conclusion of the transaction. How does the neoclassical split based on self-interest proceed?

Splitting the pie, based on neoclassical price theory

The neoclassical solution for the transaction price p is:

$$p=\frac{RS+CD}{S+D} ,$$

where each side gets the following shares:

Buyer's share: $R-p=\frac{D}{S+D}\cdot(R-C)$

Seller's share: $p-C=\frac{S}{S+D}\cdot(R-C)$

Neoclassical theory thus suggests that the two sides will agree on a price that is a *weighted average* of their reservation prices. This leads to a buyer's share of the gain from trade (R – C) that is weighted by D, representing his reluctance to buy. On the other hand, the seller's share is weighted by S, representing his reluctance to sell. The reservation prices determine the efficiency of the transaction, and the price its fairness.

Neoclassical price theory does not necessarily lead to a winner-take-all type of split that is commonly observed today in modern markets. Its starting point is a potentially even split modified by a weighing system in favor of the side that is more reluctant to conclude the transaction. The neoclassical model does not capture very well the non-deterministic nature of the bargaining process and the option to abort the transaction. It makes clear, however, that in the end, the side with the greater negotiating power (expressed as a reluctance to conclude the transaction) gets a larger share of the pie.

Splitting the pie, based on game theory

The price negotiations also remind us of the ultimatum game that so befuddles economists because its experimental results invariably go against their established theories about *Homo economicus* and self-interest. In this well-studied game, two players are handed an amount like ten dollars. If they can agree how to split it, it is theirs to split; if not, they have to return the ten dollars. The classic ultimatum game is a one-round affair—no bargaining. One side makes an offer, and the other side accepts or rejects. Multi-round ultimatum games have also been studied.

The transaction between the buyer and the seller *is* in fact an ultimatum game. The side who makes the first price offer essentially proposes how to split the gains from trade—the pie, so to speak—between them. If they cannot agree, no exchange occurs and the gains from trade are lost to both.

Bargaining has been extensively studied in game theory. Based on its assumption that a rational person is guided purely by self-interest, game theory has concluded that:

- Under one round of bargaining (equivalent to the ultimatum game), the offerer is expected to offer the least amount to the second player, who is expected to accept. This theoretical result has been contradicted in hundreds of actual ultimatum game experiments, which show considerations of fairness consistently intruding in the offers and responses of most participants. If fairness is an explicit criterion in the split, game theory suggests an even split.

- Under many rounds of bargaining, very patient players will agree to split the pie equally between them. Here, fairness clearly dominates even among self-interested parties.

Splitting the pie, based on philosophical theories of justice

In *The Political Economy of Fairness*, Zajac discusses the philosophical foundations of sharing and fairness, based on several widely differing social philosophies—1) Rawls' protection of the weakest through social institutions; 2) Nozick's minimal state and the primacy of the individual; 3) Harsanyi, a modern representative of the utilitarians and their "greatest good for the greatest number;" and 4) Varian's superfairness theory based on an envy-free endowment of wealth at birth, followed by the operation of the free market. All four adopt the even split as a generally-agreed upon starting point, though they differ in their details and expected outcomes. None of the above propose a winner-take-all approach in splitting a pie.

Splitting the pie, based on our deely-held values

Splitting the gains from trade boils down to an extremely important question that goes beyond economics: if two people are jointly given anything — a pie, for instance, a hundred pesos, or their joint gain from trade — how should they split it between the two of them? This question at the heart of every market transaction turns out to be very deep, because the answer reveals some of the deepest values of the society within which the transaction occurs.

Ask representative members of any society in existence today, from the oldest to the most recent, from the least to the most industrialized: if two of you were given a pie, how would you split it between you two? It would be safe to say that most of those asked, even if they did not represent the best values of their society, will *never* reply that they should take most of the pie themselves and just leave some crumbs for the other person. Why? Because practically all societies put a high value on fairness, justice, equity and related concepts. Societies will inculcate a strong sense of these values in all their members, as soon as they are old enough to understand them.

Fairness in sharing the gains from trade

We have now looked at the problem of sharing the gains from trade, taking several different perspectives: the mathematical solution to the neoclassical market exchange model, the solutions of game theory to fair-division problems, four social philosophies of justice, and the deepest values of most societies today. Every market transaction, it turns out, is a value-laden act at its core, a declaration of one's ideology, so to speak. Where self-interest is the dominant ideology, the split is weighted towards the more powerful side but considerations of fairness will keep intruding. Otherwise, fairness suggests an even split as a general rule.

Consider the profound implications of the recognition that at the heart of every market exchange is a reaffirmation of a society's values about sharing and fairness, and if this value-system were actually explicitly reflected in the rules of the market and the guiding principles of market players themselves. Consider the implications if *Homo economicus* members let their sense of fairness temper their self-interest, and if our laws required the biggest market players of all, the business firms and corporations, to internalize this value in their charters and by-laws.